4.41 流体運動方程式に関する一考察

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A Consideration on the Equations of Fluid Motion

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Synopsis

Navier-Stokes' equations of fluid motion are derived from the law which holds only for the small displacement. However large displacements will be observed in the neighbourhood of a body moving with high velocity and the displacements vary widely in the boundary layer. So we have derived generalized equations of fluid motion, assumming that the principal stresses are proportional to the strains by a factor and the factor and the directions of principal axes of stress vary with the place and the time. The equations are reduced to the form of zero dimension and solved, supposing that the motion of ideal fluid and one of the equations given on page 33 can be taken as the 1st approximation of the flow outside of the boundary layer and velocity distribution in the boundary layer respectively. By this solution other velocity components and the pressure can be obtained as polynomials of the square of the width of bouudary layer. The method of successive approximation will be convenient for the solutions of motion of compressible fluid and heat transfer problem.

I. 緒　言

粘性流体の運動を支配するナビヤストークスの流体運動方程式は微小変位の弾性体に対する仮定から出発して導き出されている。変位の大きい場所がある高速の場合への応用には疑問がある。簡単な仮定から出発した微分方程式であるのに係らずその解を得ることは非常に困難である。慣性項が省略出来る場合，変動速度が一定の変位速度に対して小である場合など流体運動方程式が線型微分方程式となる場合，u, v, w が適当に与えられる場合に解が求められる。流線形に近い物体周りの流は物体表面に極く近いところを除けば理想流体の流とほとんど変らない。理想流体では物体表面で滑りがあるが粘性流体の場合には物体表面の速

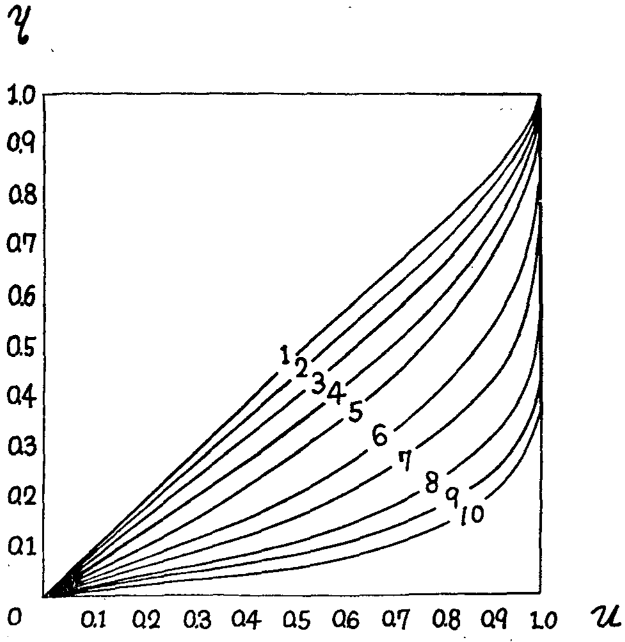
度は零である。プラントルが考え出したように粘性の影響は物体表面に沿う薄い境界層の内だけに限つてあるものと考えても第一次近似的にはよい。境界層内の速度分布は図に示されるように速度によつて変化する。この速度分布は流に平行に置かれた平板の場合に対し平板の表面で速度零，境界層の境界条件を満足するようにして求めたものである。

$\bar{y}(x,y,t)$ を境界層の厚さ，$\eta=y/\bar{y}$，$\bar{\eta}=1-\eta$，u' を x 方向の分速度，V を無限遠における流体の速度とし $u=u'/V$ とすれば境界条件から速度分布

1. $u=17\eta/16-\eta^{17}/16$
2. $u=9\eta/8-\eta^{9}/8$
3. $u=5\eta/4-\eta^{5}/4$
4. $u=3\eta/2-\eta^{3}/2$
5. $u=1-\bar{\eta}^{2}$
6. $u=1-\bar{\eta}^{3}$
7. $u=1-\bar{\eta}^{4}$
8. $u=1-\bar{\eta}^{6}$
9. $u=1-\bar{\eta}^{8}$
10. $u=1-\bar{\eta}^{10}$

が得られる。

これらは Burgers, Nikuradze らの実験結果とよく一致する。



図

このことから粘性流体の物体周りの流は第1次近似としては理想流体の流を物体表面に沿つて境界層の厚さ $\bar{y}$ だけ y 方向に移動し境界層内は上に与えた速度分布の中マツハ数 M の大さに対し適当なものをとつて得られるものと考えてよいと思われる。境界層外の流に対しては速度，圧力は $u(x,y-\bar{y},z,t)$，$v(x,y-\bar{y},z,t)$，$w(x,y-\bar{y},z,t)$ $p(x,y-\bar{y},z,t)$ で与えられるものとしその第1次近似は境界層のない理想流体の流から得られる値をとるものとする。

図から明かなように速度のおそいときは境界層の大部分で $\dfrac{\partial u}{\partial y}$ は一定であるが速度が増すと共に $\dfrac{\partial u}{\partial y}=\text{const}$ の部分が段々減少していく。$\dfrac{\partial u}{\partial y}$ が一定でない部分に対し変位と応力が一定の比率で比例すると考えるのは無理ではないかと思われる。図では零デメンションで表わされているから変位の大さの変化は余り大きくないように見えるが実際は速度が掛かるので変化の範囲は相当広い。 Blasius などの境界層方程式の解から分るようにナビヤストークスの流体運動方程式から出発した結果では $\dfrac{\partial u}{\partial y}=\text{const}$ の範囲だけに対する解が得られる。従つて速度の速い乱流境界層に対してはほとんどその底層にだけ適用出来る解しか得られな

い。他の部分に対しては上の比例定数が変化するものと考えるべきではないか。次に応力を導き出すとき主応力方向の変化が考慮されていないがある場合にはこれを考慮する必要があるように思われる。IIでこれらのことを考慮して一般化された流体運動方程式を導き出した。

これを零デメンションの形に書き直しこれを使つて流に平行に置かれた平板の場合に対する解法の概略をIIIで述べた。

この方法によれば u, v, w, p, μ_1 は $\bar{y}^2$ の多項式で表わされる。

今までの境界層微分方程式から得られる解はこの方法の第1次近似の一部分で境界層の境界附近では差異がある。境界層の境界附近の速度分布はこの方法によるものの方が実際とよく一致する。

粘性係数に場所と時間の函数が掛つた粘性項が入つて運動方程式は複雑になつたが境界層の速度分布を予め与えることによつて微分方程式の解法は却つて楽になる。

境界層内のある部分では従来の関係が成立つ。

ナビヤ　ストークスの流体運動方程式を使つて得られた解で実際とよく一致するものにTaylorの回転同心円柱間の運動がある。これは境界層外の流が境界層内の流に支配され渦流となるため上のような境界層の境界での条件が必要でなくなつているためと思われる。

この方法によれば乱流境界層の場合も平均値をとつた流体運動方程式でなく時間的変動を考慮に入れて解くことが出来る。また圧縮性の影響,温度の影響を考えるのにも便利である。

II. 流体運動方程式の一般化

$Oxyz$ を直角座標，$OXYZ$ を主応力方向にとつた直角座標とし方向余弦が

	X	Y	Z
x	l_1	l_2	l_3
y	m_1	m_2	m_3
z	n_1	n_2	n_3

で与えられるような関係にあるものとする。u, v, w を速度の x, y, z 方向の分速度，p を圧力，p_{xx}, p_{yy}, p_{zz}, $p_{xy}=p_{yx}$, $p_{xz}=p_{zx}$, $p_{yz}=p_{zy}$ を応力，U, V, W を速度の X, Y, Z 方向の分速度，p_1, p_2, p_3 を X, Y, Z 方向の主応力とする。

μ_1, μ_2 を x, y, z, t の函数，μ を粘性係数とし

$$p_1=-p+\mu\left\{2\mu_1\frac{\partial U}{\partial X}+\mu_2\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}+\frac{\partial W}{\partial Z}\right)\right\},$$

$$p_2=-p+\mu\left\{2\mu_1\frac{\partial V}{\partial Y}+\mu_2\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}+\frac{\partial W}{\partial Z}\right)\right\},$$

$$p_3=-p+\mu\left\{2\mu_1\frac{\partial W}{\partial Z}+\mu_2\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}+\frac{\partial W}{\partial Z}\right)\right\}$$

とする。

然るときは

$$p_{xx}=l_1^2p_1+l_2^2p_2+l_3^2p_3,\ p_{yy}=m_1^2p_1+m_2^2p_2+m_3^2p_3,$$

$$p_{zz}=n_1^2p_1+n_2^2p_2+m_3^2p_3,\ p_{xy}=l_1m_1p_1+l_2m_2p_2+l_3m_3p_3,$$

$$p_{xz}=n_1l_1p_1+n_2l_2p_2+n_3l_3p_3,\ p_{yz}=m_1n_1p_1+m_2n_2p_2+m_3n_3p_3,$$

$$U=l_1u+m_1v+n_1w,\ V=l_2u+m_2v+n_2w,$$

$$W=l_3u+m_3v+n_3w,$$

$$\frac{\partial}{\partial X}=l_1\frac{\partial}{\partial x}+m_1\frac{\partial}{\partial y}+n_1\frac{\partial}{\partial z},\quad \frac{\partial}{\partial Y}=l_2\frac{\partial}{\partial x}+m_2\frac{\partial}{\partial y}+n_2\frac{\partial}{\partial z},$$

$$\frac{\partial}{\partial Z}=l_3\frac{\partial}{\partial x}+m_3\frac{\partial}{\partial y}+n_3\frac{\partial}{\partial z},$$

$$l_1^2+l_2^2+l_3^2=1,\quad m_1^2+m_2^2+m_3^2=1,\quad n_1^2+n_2^2+n_3^2=1$$

$$l_1m_1+l_2m_2+l_3m_3=0\quad m_1n_1+m_2n_2+m_3n_3=0$$

$$n_1l_1+n_2l_2+n_3l_3=0$$

であるから

$$p_{xx}=-p+\mu\left\{2\mu_1\frac{\partial u}{\partial x}+\mu_2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\right\},$$

$$p_{yy}=-p+\mu\left\{2\mu_1\frac{\partial v}{\partial y}+\mu_2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\right\},$$

$$p_{zz}=-p+\mu\left\{2\mu_1\frac{\partial w}{\partial z}+\mu_2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\right\}$$

となる。

$$L_m=m_1\frac{\partial l_1}{\partial x}+m_2\frac{\partial l_2}{\partial x}+m_3\frac{\partial l_3}{\partial x},\quad M_l=l_1\frac{\partial m_1}{\partial y}+l_2\frac{\partial m_2}{\partial y}+l_3\frac{\partial m_3}{\partial y},$$

$$M_n=n_1\frac{\partial m_1}{\partial y}+n_2\frac{\partial m_2}{\partial y}+n_3\frac{\partial m_3}{\partial y},\quad N_m=m_1\frac{\partial n_1}{\partial z}+m_2\frac{\partial n_2}{\partial z}+m_3\frac{\partial n_3}{\partial z}$$

$$N_l=l_1\frac{\partial n_1}{\partial z}+l_2\frac{\partial n_2}{\partial z}+l_3\frac{\partial n_3}{\partial z},\quad L_n=n_1\frac{\partial l_1}{\partial x}+n_2\frac{\partial l_2}{\partial x}+n_3\frac{\partial l_3}{\partial x}$$

とすれば同様にして

$$p_{xy}=\mu\mu_1\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}+L_mu+M_lv\right),\quad p_{yz}=\mu\mu_1\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}+M_nv+N_mw\right)$$

$$p_{zx}=\mu\mu_1\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}+N_lw+L_nu\right)$$

となる。

これらを流体運動方程式

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\frac{\partial p_{xx}}{\partial x}+\frac{\partial p_{xy}}{\partial y}+\frac{\partial p_{xz}}{\partial z}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\frac{\partial p_{xy}}{\partial x}+\frac{\partial p_{yy}}{\partial y}+\frac{\partial p_{yz}}{\partial z}$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=\frac{\partial p_{xy}}{\partial x}+\frac{\partial p_{yz}}{\partial y}+\frac{\partial p_{zz}}{\partial z}$$

に代入すれば

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)+\frac{\partial p}{\partial x}=\mu\left[2\frac{\partial}{\partial x}\left(\mu_1\frac{\partial u}{\partial x}\right)\right.$$

$$+\frac{\partial}{\partial y}\left\{\mu_1\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}+L_m u+M_e v\right)\right\}+\frac{\partial}{\partial z}\left\{\mu_1\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right.\right.$$

$$\left.\left.\left.+N_l w+L_n u\right)\right\}+\frac{\partial}{\partial x}\left\{\mu_2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\right\}\right],$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)+\frac{\partial p}{\partial y}=\mu\left[\frac{\partial}{\partial x}\left\{\mu_1\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right.\right.\right.$$

$$\left.\left.+L_m u+M_l v\right)\right\}+2\frac{\partial}{\partial y}\left(\mu_1\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left\{\mu_1\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right.\right.$$

$$\left.\left.\left.+M_n v+N_m w\right)\right\}+\frac{\partial}{\partial y}\left\{\mu_2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\right\}\right],$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)+\frac{\partial p}{\partial z}=\mu\left[\frac{\partial}{\partial x}\left\{\mu_1\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right.\right.\right.$$

$$\left.\left.+N_l w+L_n u\right)\right\}+\frac{\partial}{\partial y}\left\{\mu_1\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}+M_n v+N_m w\right)\right\}$$

$$\left.+2\frac{\partial}{\partial z}\left(\mu_1\frac{\partial w}{\partial z}\right)+\frac{\partial}{\partial z}\left\{\mu_2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\right\}\right]$$

となる。

aを音速とすれば，$\frac{\partial\rho}{\partial p}=1/a^2$ であるから連続の方程式は

$$\rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=-\left(\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}\right)\Big/a^2$$

となる。

$$x=Lx',\quad y=Ly',\quad z=Bz/2,\quad t=Lt'/V,\quad u=Vu',\quad v=Vv',\quad w=Vw',$$

$$p=\rho V^2p',\quad M=V/a,\quad l=2L/B,\quad R=\rho VL/\mu,$$

$$L'_m=m_1\frac{\partial l_1}{\partial x'}+m_2\frac{\partial l_2}{\partial x'}+m_3\frac{\partial l_3}{\partial x'},\quad M_l'=l_1\frac{\partial m_1}{\partial y'}+l_2\frac{\partial m_2}{\partial y'}+l_3\frac{\partial m_3}{\partial y'},$$

$$M_n'=n_1\frac{\partial m_1}{\partial y'}+n_2\frac{\partial m_2}{\partial y'}+n_3\frac{\partial m_3}{\partial y'},\quad N_m'=m_1\frac{\partial n_1}{\partial z'}+m_2\frac{\partial n_2}{\partial z'}+m_3\frac{\partial n_3}{\partial z'}$$

$$N_l'=l_1\frac{\partial n_1}{\partial z'}+l_2\frac{\partial n_2}{\partial z'}+l_3\frac{\partial n_3}{\partial z'},\quad L_n'=n_1\frac{\partial l_1}{\partial x'}+n_2\frac{\partial l_2}{\partial x'}+n_3\frac{\partial l_3}{\partial x'}$$

としダツシユを省略すれば

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+l\frac{\partial w}{\partial z}=-M^2\left(\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}\right)\Big/(1-pM^2)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+lw\frac{\partial u}{\partial z}+\frac{\partial p}{\partial x}\Big/(1-pM^2)=\left[2\frac{\partial}{\partial x}\left(\mu_1\frac{\partial u}{\partial x}\right)\right.$$

$$+\frac{\partial}{\partial y}\left\{\mu_1\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}+L_m u+M_l v\right)\right\}+l\frac{\partial}{\partial z}\left\{\mu_1\left(l\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right.\right.$$

$$\left.\left.\left.+N_l w+L_n u\right)\right\}+\frac{\partial}{\partial x}\left\{\mu_2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+l\frac{\partial w}{\partial z}\right)\right\}\right]\Big/R,$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+lw\frac{\partial v}{\partial z}+\frac{\partial p}{\partial y}\bigg/(1-pM^2)=\left[\frac{\partial}{\partial x}\left\{\mu_1\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right.\right.\right.$$
$$\left.\left.+L_m u+M_l v\right)\right\}+2\frac{\partial}{\partial y}\left(\mu_1\frac{\partial v}{\partial y}\right)+l\frac{\partial}{\partial z}\left\{\mu_1\left(\frac{\partial w}{\partial y}+l\frac{\partial v}{\partial z}+M_n v+N_m w\right)\right\}$$
$$\left.+\frac{\partial}{\partial y}\left\{\mu_2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+l\frac{\partial w}{\partial z}\right)\right\}\right]\bigg/R,$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+lw\frac{\partial w}{\partial z}+l\frac{\partial p}{\partial z}\bigg/(1-pM^2)=\left[\frac{\partial}{\partial x}\left\{\mu_1\left(l\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right.\right.\right.$$
$$\left.\left.+N_l w+L_n u\right)\right\}+\frac{\partial}{\partial y}\left\{\mu_1\left(\frac{\partial w}{\partial y}+l\frac{\partial v}{\partial z}+M_n v+N_m w\right)\right\}$$
$$\left.+2l^2\frac{\partial}{\partial z}\left(\mu_1\frac{\partial w}{\partial z}\right)+l\frac{\partial}{\partial z}\left\{\mu_2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+l\frac{\partial w}{\partial z}\right)\right\}\right]\bigg/R$$

となる。

流に平行に置かれた平板の場合以外に対してはこれを更に直交曲線座標に変換する。

III. 一般化された流体運動式の解

流に平行に置かれた平板の場合について考える。

$$L_m=L_n=M_l=M_u=N_l=M_m=0,\quad u_2=-2\mu_1/3$$

とし

$$u=u_1+u_2',\quad v=v_1+v_2',\quad w=w_1+w_2',\quad p=p_1+p_2',\quad \mu_1=\mu_{11}+\mu_{12}'$$

と置く。

1) 境界層外の流に対し

$$u_1=1,\quad v_1=w_1=p_1=\mu_{11}=0$$

とすれば連続の式および流体運動方程式は

$$\frac{\partial u_2'}{\partial x}-\frac{\partial u_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}+\frac{\partial v_2'}{\partial y}+l\left(\frac{\partial w_2'}{\partial z}-\frac{\partial w_2'}{\partial y}\frac{\partial \bar{y}}{\partial z}\right)=-M^2\left\{\frac{\partial p_2'}{\partial t}-\frac{\partial p_2'}{\partial y}\frac{\partial \bar{y}}{\partial t}+\frac{\partial p_2'}{\partial x}-\right.$$
$$\left.\frac{\partial p_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}+u_2'\left(\frac{\partial p_2'}{\partial x}-\frac{\partial p_2}{\partial y}\frac{\partial \bar{y}}{\partial x}\right)+v_2'\frac{\partial p_2'}{\partial y}+lw_2'\left(\frac{\partial p_2'}{\partial z}-\frac{\partial p_2'}{\partial y}\frac{\partial \bar{y}}{\partial z}\right)\right\}\bigg/$$
$$(1-p_2'M^2) \tag{1}$$

$$\frac{\partial u_2'}{\partial t}-\frac{\partial u_2'}{\partial y}\frac{\partial \bar{y}}{\partial t}+\frac{\partial u_2'}{\partial x}-\frac{\partial u_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}+u_2'\left(\frac{\partial u_2'}{\partial x}-\frac{\partial u_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}\right)+v_2'\frac{\partial u_2'}{\partial y}$$
$$+lw_2'\left(\frac{\partial u_2'}{\partial z}-\frac{\partial u_2'}{\partial y}\frac{\partial \bar{y}}{\partial z}\right)+\left(\frac{\partial p_2'}{\partial x}-\frac{\partial p_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}\right)\bigg/(1-p_2'M^2)$$
$$=\left[2\frac{\partial}{\partial x}\left\{\mu_{12}'\left(\frac{\partial u_2'}{\partial x}-\frac{\partial u_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}\right)\right\}-2\frac{\partial}{\partial y}\left\{\mu_{12}'\left(\frac{\partial u_2'}{\partial x}-\frac{\partial u_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}\right)\right\}\frac{\partial \bar{y}}{\partial x}\right.$$
$$+\frac{\partial}{\partial y}\left\{\mu_{12}'\left(\frac{\partial v_2'}{\partial x}-\frac{\partial v_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}+\frac{\partial u_2'}{\partial y}\right)\right\}+l\frac{\partial}{\partial z}\left[\mu_{12}'\left\{l\left(\frac{\partial u_2'}{\partial z}-\frac{\partial u_2'}{\partial y}\frac{\partial \bar{y}}{\partial z}\right)\right.\right.$$
$$\left.\left.+\frac{\partial w_2'}{\partial x}-\frac{\partial w_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}\right\}\right]-l\frac{\partial}{\partial y}\left[\mu_{12}'\left\{l\left(\frac{\partial u_2'}{\partial z}-\frac{\partial u_2'}{\partial y}\frac{\partial \bar{y}}{\partial z}\right)+\frac{\partial w_2'}{\partial x}\right.\right.$$

$$-\frac{\partial w_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}\Big)\Big\}\Big]\frac{\partial \bar{y}}{\partial z}-2\frac{\partial}{\partial x}\Big[\mu_{12}'\Big\{\frac{\partial u_2'}{\partial x}-\frac{\partial u_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}+\frac{\partial v_2'}{\partial y}+l\Big(\frac{\partial w_2'}{\partial z}$$

$$-\frac{\partial w_2'}{\partial y}\frac{\partial \bar{y}}{\partial z}\Big)\Big\}\Big]\Big/3+2\frac{\partial}{\partial y}\Big[\mu_{12}'\Big\{\frac{\partial u_2'}{\partial x}-\frac{\partial u_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}+\frac{\partial v_2'}{\partial y}+l\Big(\frac{\partial w_2'}{\partial z}$$

$$-\frac{\partial w_2'}{\partial y}\frac{\partial \bar{y}}{\partial z}\Big)\Big\}\Big]\frac{\partial \bar{y}}{\partial x}\Big/3\Big]\Big/R \qquad 2),$$

$$\frac{\partial v_2'}{\partial t}-\frac{\partial v_2'}{\partial y}\frac{\partial \bar{y}}{\partial t}+\frac{\partial v_2'}{\partial x}-\frac{\partial v_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}+u_2'\Big(\frac{\partial v_2'}{\partial x}-\frac{\partial v_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}\Big)+v_2'\frac{\partial v_2'}{\partial y}$$

$$+lw_2'\Big(\frac{\partial v_2'}{\partial z}-\frac{\partial v_2'}{\partial y}\frac{\partial \bar{y}}{\partial z}\Big)+\frac{\partial p_2'}{\partial y}\Big/(1-p_2'M^2)=\Big[\frac{\partial}{\partial x}\Big\{\mu_{12}'\Big(\frac{\partial v_2'}{\partial x}$$

$$-\frac{\partial v_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}\Big)+\frac{\partial u_2'}{\partial y}\Big)\Big\}-\frac{\partial}{\partial y}\Big\{\mu_{12}'\Big(\frac{\partial v_2'}{\partial x}-\frac{\partial v_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}+\frac{\partial u_2'}{\partial y}\Big)\Big\}\frac{\partial \bar{y}}{\partial x}$$

$$+2\frac{\partial}{\partial y}\Big(\mu_{12}'\frac{\partial v_2'}{\partial y}\Big)+l\frac{\partial}{\partial z}\Big[\mu_{12}'\Big\{\frac{\partial w_2'}{\partial y}+l\Big(\frac{\partial v_2'}{\partial z}-\frac{\partial v_2'}{\partial y}\frac{\partial \bar{y}}{\partial z}\Big)\Big\}\Big]$$

$$-l\frac{\partial}{\partial y}\Big[\mu_{12}'\Big\{\frac{\partial w_2'}{\partial y}+l\Big(\frac{\partial v_2'}{\partial z}-\frac{\partial v_2'}{\partial y}\frac{\partial \bar{y}}{\partial z}\Big)\Big\}\Big]\frac{\partial \bar{y}}{\partial z}-2\frac{\partial}{\partial y}\Big[\mu_{12}'\Big\{\frac{\partial u_2'}{\partial x}$$

$$-\frac{\partial u_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}+\frac{\partial v_2'}{\partial y}+l\Big(\frac{\partial w_2'}{\partial z}-\frac{\partial w_2'}{\partial y}\frac{\partial \bar{y}}{\partial z}\Big)\Big\}\Big]\Big/3\Big]\Big/R \qquad 3),$$

$$\frac{\partial w_2'}{\partial t}-\frac{\partial w_2'}{\partial y}\frac{\partial \bar{y}}{\partial t}+\frac{\partial w_2'}{\partial x}-\frac{\partial w_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}+u_2'\Big(\frac{\partial w_2'}{\partial x}-\frac{\partial w_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}\Big)+v_2'\frac{\partial w_2'}{\partial y}$$

$$+lw_2'\Big(\frac{\partial w_2'}{\partial z}-\frac{\partial w_2'}{\partial y}\frac{\partial \bar{y}}{\partial z}\Big)+l\Big(\frac{\partial p_2'}{\partial z}-\frac{\partial p_2'}{\partial y}\frac{\partial \bar{y}}{\partial z}\Big)\Big/(1-p_2'M^2)$$

$$=\Big[\frac{\partial}{\partial x}\Big[\mu_{12}'\Big\{l\Big(\frac{\partial u_2'}{\partial z}-\frac{\partial u_2'}{\partial y}\frac{\partial \bar{y}}{\partial z}\Big)+\frac{\partial w_2'}{\partial x}-\frac{\partial w_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}\Big\}\Big]$$

$$-\frac{\partial}{\partial y}\Big[\mu_{12}'\Big\{l\Big(\frac{\partial u_2'}{\partial z}-\frac{\partial u_2'}{\partial y}\frac{\partial \bar{y}}{\partial z}\Big)+\frac{\partial w_2'}{\partial x}-\frac{\partial w_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}\Big\}\Big]\frac{\partial \bar{y}}{\partial x}$$

$$+\frac{\partial}{\partial y}\Big[\mu_{12}'\Big\{\frac{\partial w_2'}{\partial y}+l\Big(\frac{\partial v_2'}{\partial z}-\frac{\partial v_2'}{\partial y}\frac{\partial \bar{y}}{\partial z}\Big)\Big\}\Big]+2l^2\frac{\partial}{\partial z}\Big\{\mu_{12}'\Big(\frac{\partial w_2'}{\partial z}$$

$$-\frac{\partial w_2'}{\partial y}\frac{\partial \bar{y}}{\partial z}\Big)\Big\}-2l^2\frac{\partial}{\partial y}\Big\{\mu_{12}'\Big(\frac{\partial w'}{\partial z}-\frac{\partial w_2'}{\partial y}\frac{\partial \bar{y}}{\partial z}\Big)\Big\}\frac{\partial \bar{y}}{\partial z}-2l\frac{\partial}{\partial z}\Big[\mu_{12}'\Big\{\frac{\partial u_2'}{\partial x}$$

$$-\frac{\partial u_2'}{\partial y}\frac{\partial \bar{y}}{\partial y}+\frac{\partial v_2'}{\partial y}+l\Big(\frac{\partial w_2'}{\partial z}-\frac{\partial w_2'}{\partial y}\frac{\partial \bar{y}}{\partial z}\Big)\Big\}\Big]\Big/3+2l\frac{\partial}{\partial y}\Big[\mu_{12}'\Big\{\frac{\partial u_2'}{\partial x}$$

$$-\frac{\partial u_2'}{\partial y}\frac{\partial \bar{y}}{\partial x}+\frac{\partial v_2'}{\partial y}+l\Big(\frac{\partial w_2'}{\partial z}-\frac{\partial w_2'}{\partial y}\frac{\partial \bar{y}}{\partial z}\Big)\Big\}\Big]\frac{\partial \bar{y}}{\partial z}\Big/3\Big]\Big/R \qquad 4),$$

となる。

2）境界層内の流に対しては

$$\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial y}=0 \qquad 5),$$

$$\frac{\partial u_2'}{\partial x}+\frac{\partial v_2'}{\partial y}+l\Big(\frac{\partial w_1}{\partial z}+\frac{\partial w_2'}{\partial z}\Big)=-M^2\Big\{\frac{\partial p_1}{\partial t}+\frac{\partial p_2'}{\partial t}$$

$$+(u_1+u_2')\Big(\frac{\partial p_1}{\partial x}+\frac{\partial p_2'}{\partial x}\Big)+(v_1+v_2')\Big(\frac{\partial p_1}{\partial y}+\frac{\partial p_2'}{\partial y}\Big)$$

$$+l(w_1+w_2')\Big(\frac{\partial p_1}{\partial z}+\frac{\partial p_2'}{\partial z}\Big)\Big\}\Big/(1-\overline{p_1+p_2}'M^2) \qquad 6),$$

$$\frac{\partial u_1}{\partial t}+u_1\frac{\partial u_1}{\partial x}+v_1\frac{\partial u_1}{\partial y}=\frac{\partial}{\partial y}\left(\mu_{11}\frac{\partial u_1}{\partial y}\right)\Big/R \qquad 7),$$

$$\begin{aligned}&\frac{\partial u_2'}{\partial t}+u_2'\frac{\partial u_1}{\partial x}+(u_1+u_2')\frac{\partial u_2'}{\partial x}+v_2'\frac{\partial u_1}{\partial y}+(v_1+v_2')\frac{\partial u_2'}{\partial y}+l(w_1+w_2')\left(\frac{\partial u_1}{\partial z}\right.\\&\quad\left.+\frac{\partial u_2'}{\partial z}\right)+\left(\frac{\partial p_1}{\partial x}+\frac{\partial p_2'}{\partial x}\right)\Big/(1-\overline{p_1+p_2'}M^2)=\left[2\frac{\partial}{\partial x}\left\{(\mu_{11}+\mu_{12}')\left(\frac{\partial u_1}{\partial x}\right.\right.\right.\\&\quad\left.\left.+\frac{\partial u_2'}{\partial x}\right)\right\}+\frac{\partial}{\partial y}\left\{\mu_{12}'\frac{\partial u_1}{\partial y}+(\mu_{11}+\mu_{12}')\left(\frac{\partial v_1}{\partial x}+\frac{\partial v_2'}{\partial x}+\frac{\partial u_2'}{\partial y}\right)\right\}\\&\quad+l\frac{\partial}{\partial z}\left\{(\mu_{11}+\mu_{12}')\left(l\frac{\partial u_1}{\partial z}+l\frac{\partial u_2'}{\partial z}+\frac{\partial w_1}{\partial x}+\frac{\partial w_2'}{\partial x}\right)\right\}\\&\quad\left.-2\frac{\partial}{\partial x}\left\{(\mu_{11}+\mu_{12}')\left(\frac{\partial u_2'}{\partial x}+\frac{\partial v_2'}{\partial y}+l\frac{\partial w_1}{\partial z}+l\frac{\partial w_2'}{\partial z}\right)\right\}\Big/3\right]\Big/R \qquad 8),\end{aligned}$$

$$\frac{\partial v_1}{\partial t}+u_1\frac{\partial v_1}{\partial x}+v_1\frac{\partial v_1}{\partial y}+\frac{\partial p_1}{\partial y}=\left[\frac{\partial}{\partial x}\left(\mu_{11}\frac{\partial u_1}{\partial y}\right)+2\frac{\partial}{\partial y}\left(\mu_{11}\frac{\partial v_1}{\partial y}\right)\right]\Big/R \qquad 9),$$

$$\begin{aligned}&\frac{\partial v_2'}{\partial t}+u_2'\frac{\partial v_1}{\partial x}+(u_1+u_2')\frac{\partial v_2'}{\partial x}+v_2'\frac{\partial v_1}{\partial y}+(v_1+v_2')\frac{\partial v_2'}{\partial y}+l(w_1+w_2')\left(\frac{\partial v_1}{\partial z}\right.\\&\quad\left.+\frac{\partial v_2'}{\partial z}\right)+(p_1+p_2')M^2\frac{\partial p}{\partial y}\Big/(1-\overline{p_1+p_2'}M^2)+\frac{\partial p_2'}{\partial y}\Big/(1-\overline{p_1+p'_2}M^2)\\&\quad=\left[\frac{\partial}{\partial x}\left\{\mu_{12}'\frac{\partial u_1}{\partial y}+(\mu_{11}+\mu_{12}')\left(\frac{\partial v_1}{\partial x}+\frac{\partial v_2'}{\partial x}+\frac{\partial u_2'}{\partial y}\right)\right\}+2\frac{\partial}{\partial y}\left\{\mu_{12}'\frac{\partial v_1}{\partial y}\right.\right.\\&\quad\left.+(\mu_{11}+\mu_{12}')\frac{\partial v_2'}{\partial y}\right\}+l\frac{\partial}{\partial z}\left[(\mu_{11}+\mu_{12}')\left\{\frac{\partial w_1}{\partial y}+\frac{\partial w_2'}{\partial y}+l\left(\frac{\partial v_1}{\partial z}+\frac{\partial v_2'}{\partial z}\right)\right\}\right]\\&\quad\left.-2\frac{\partial}{\partial y}\left\{(\mu_{11}+\mu_{12}')\left(\frac{\partial u_2'}{\partial x}+\frac{\partial v_2'}{\partial y}+l\frac{\partial w_1}{\partial z}+l\frac{\partial w_2'}{\partial z}\right)\right\}\Big/3\right]\Big/R \qquad 10),\end{aligned}$$

$$\begin{aligned}&\frac{\partial w_1}{\partial t}+u_1\frac{\partial w_1}{\partial x}+v_1\frac{\partial w_1}{\partial y}+l\frac{\partial p_1}{\partial z}=\left[l\frac{\partial}{\partial x}\left(\mu_{11}\frac{\partial u_1}{\partial z}\right)\right.\\&\quad\left.+\frac{\partial}{\partial y}\left\{\mu_{11}\left(\frac{\partial w_1}{\partial y}+l\frac{\partial v_1}{\partial z}\right)\right\}\right]\Big/R \qquad 11),\end{aligned}$$

$$\begin{aligned}&\frac{\partial w_2'}{\partial t}+u_2'\frac{\partial w_1}{\partial x}+(u_1+u_2')\frac{\partial w_2'}{\partial x}+v_2'\frac{\partial w_1}{\partial y}+(v_1+v_2')\frac{\partial w_2'}{\partial y}+l(w_1+w_2')\left(\frac{\partial w_1}{\partial z}\right.\\&\quad\left.+\frac{\partial w_2'}{\partial z}\right)+l(p_1+p_2')M^2\frac{\partial p_1}{\partial z}\Big/(1-\overline{p_1+p_2'}M^2)+l\frac{\partial p_2'}{\partial z}\Big/(1-\overline{p_1+p_2'}M^2)\\&\quad=\left[\frac{\partial}{\partial x}\left\{l\mu_{12}'\frac{\partial u_1}{\partial z}+(\mu_{11}+\mu_{12}')\left(l\frac{\partial u_2'}{\partial z}+\frac{\partial w_1}{\partial z}+\frac{\partial w_2'}{\partial z}\right)\right\}\right.\\&\quad+\frac{\partial}{\partial y}\left\{\mu_{12}'\left(\frac{\partial w_1}{\partial y}+l\frac{\partial v_1}{\partial z}+(\mu_{11}+\mu_{12}')\left(\frac{\partial w_2'}{\partial y}+l\frac{\partial v_2'}{\partial z}\right)\right)\right\}\\&\quad+2l^2\frac{\partial}{\partial z}\left\{(\mu_{11}+\mu_{12}')\left(\frac{\partial w_1}{\partial z}+\frac{\partial w_2'}{\partial z}\right)\right\}-2l\frac{\partial}{\partial z}\left\{(\mu_{11}+\mu_{12}')\left(\frac{\partial u_2'}{\partial x}\right.\right.\\&\quad\left.\left.\left.+\frac{\partial v_2'}{\partial y}+l\frac{\partial w_1}{\partial z}+l\frac{\partial w_2'}{\partial z}\right)\right\}\Big/3\right]\Big/R \qquad 12),\end{aligned}$$

となる。

今 $u_1=f_1(\eta)$ とすれば 5) により

$$\frac{\partial v_1}{\partial y}=-\frac{\partial u_1}{\partial x}=f_1'(\eta)\eta\frac{\partial\bar{y}}{\partial x}\Big/\bar{y}$$

故に

$$v_1 = \frac{\partial \bar{y}}{\partial x} \int f_1'(\eta)\eta d\eta = g_1(\eta) \frac{\partial \bar{y}}{\partial x}$$

となる。

$$\frac{\partial u_1}{\partial t} = -f_1'(\eta)\eta \frac{\partial \bar{y}}{\partial t} \Big/ \bar{y}, \qquad \frac{\partial u_1}{\partial y} = f_1'(\eta)/\bar{y}$$

であるから 7) は

$$-f_1'(\eta)\eta \frac{\partial \bar{y}}{\partial t} \Big/ \bar{y} - \Big\{f_1(\eta)\eta - g_1(\eta)\Big\} f_1'(\eta) \frac{\partial \bar{y}}{\partial x} \Big/ \bar{y} = \frac{\partial}{\partial y} \Big\{\mu_{11} f_1'(\eta)/\bar{y}\Big\}/R$$

となる。これを積分すれば

$$\mu_{11} = -R\bar{y}\Big[\frac{\partial \bar{y}}{\partial t} \int f_1'(\eta)\eta d\eta + \frac{\partial \bar{y}}{\partial x} \int \Big\{f_1(\eta)\eta - g_1(\eta)\Big\} f_1'(\eta) d\eta\Big] \Big/ f_1'(\eta)$$

が得られる。

$$\tau_1(\eta) = -\int f_1'(\eta)\eta d\eta, \qquad x_1(\eta) = -\int \Big\{f_1(\eta)\eta - g_1(\eta)\Big\} f_1'(\eta) d\eta$$

$$\mu_{t11} = R\bar{y}\tau_1(\eta) \frac{\partial \bar{y}}{\partial t} \Big/ f_1'(\eta), \qquad \mu_{x11} = R\bar{y}x_1(\eta) \frac{\partial \bar{y}}{\partial x} \Big/ f_1'(\eta) \qquad 13),$$

と置けば

$$\mu_{11} = \mu_{t11} + \mu_{x11}$$

となる。

$$\zeta_0 = 1 - (z - z_0)^3/(1 - z_0)^3$$

と置き $\eta = 0$ に対し

$$\mu_{t11} = c_1 \zeta_0^{(1+\gamma)m'} x^{m+(m+1)\gamma}/\bar{y}^n, \qquad \mu_{x11} = d_1 \zeta_0^{(1+\gamma)m'} x^{m+(m+1)\gamma}/\bar{y}^n$$

と仮定すれば 13) から

$$\bar{y}^{n+1} \frac{\partial \bar{y}}{\partial t} = C_1 \zeta_0^{(1+\gamma)m'} x^{m+(m+1)\gamma}/R \qquad 14),$$

$$\bar{y}^{n+1} \frac{\partial \bar{y}}{\partial x} = D_1 \zeta_0^{(1+\gamma)m'} x^{m+(m+1)\gamma}/R \qquad 15),$$

但　$C_1 = c_1 f_1'(0)/\tau_1(0), \qquad D_1 = d_1 f_1'(0)/x_1(0)$

が得られる。

$$\bar{y} = y_{xz}(1 + Y)$$

と置き $t = 0$ で

$$Y = \frac{\partial Y}{\partial x} = 0$$

と仮定すれば 15) は

$$y_{xz}^{n+1} \frac{\partial y_{xz}}{\partial x} = D_1 \zeta_0^{(1+\gamma)m'} x^{m+(m+1)\gamma}/R$$

となる。これを積分すれば

$$y_{xz} = [(n+2)D_1/(1+\gamma)(m+1)R^{1/(n+2)}] \zeta_0^{(1+\gamma)m'/(n+2)} x^{(+\gamma)(m+1)/(n+2)}$$

が得られる。これを 14), 15) に代入すれば

$$\frac{\partial\bar{y}}{\partial t}=(1+\gamma)(m+1)C_1\bar{y}(1-\overline{n+2}Y)/(n+2)D_1x \qquad 16),$$

$$\frac{\partial\bar{y}}{\partial x}=(1+\gamma)(m+1)\bar{y}(1-\overline{n+2}Y)/(n+2)x \qquad 17),$$

となる。

16), 17) から

$$\frac{\partial Y}{\partial t}=(1+\gamma)(m+1)C_1(1-\overline{n+1}Y)/(n+2)D_1x$$

$$\frac{\partial Y}{\partial x}=-(1+\gamma)(m+1)Y/x$$

が得られ微分方程式

$$(n+2)^2D_1^2\frac{\partial^3 Y}{\partial t^3}\Big/(1+\gamma)^2(m+1)^2(n+1)^2=\left(\frac{\partial^3 Y}{\partial t\partial x^2}+\frac{\partial^2 Y}{\partial t\partial x}\Big/x\right)\Big/(1+\gamma)^2(m+1)^2$$

を得る。

$J_n(z)$, $Y_n(z)$ を第1種，第2種の n 階ベッセル函数とすれば上の微分方程式の解として

$$Y=\sum_{n=1}\left\{c_nJ_0[(1+\gamma)(m+1)\omega_nx]+d_nY_0[(1+\gamma)(m+1)\omega_nx]\right\}$$
$$\times\sin[(1+\gamma)(m+1)(n+1)\omega_nt/(n+2)D_1]$$

が得られる。

$x=x_0$ に対し $J_0[(1+\gamma)(m+1)\omega_nx]=0$

として

$$\omega_n=j_{on}/(1+\gamma)(m+1)x_0$$

となる。但 j_{on} は $J_0(z)$ の零である。

$x=\bar{x}_0$ に対し $\quad \frac{\partial Y}{\partial x}=-(1+\gamma)(m+1)Y/x$

とすれば

$$\frac{dJ_0(z)}{dz}=-J_1(z),\qquad \frac{dY_0(z)}{dz}=-Y_1(z)$$

であるから

$$d_n=-C_n\left\{\omega_n\bar{x}_0J_1[(1+\gamma)(m+1)\omega_n\bar{x}_0]-J_0[(1+\gamma)(m+1)\omega_n\bar{x}_0]\right\}$$
$$\div\left\{\omega_n\bar{x}_0Y_1[(1+\gamma)(m+1)\omega_n\bar{x}_0]-Y_0[(1+\gamma)(m+1)\omega_n\bar{x}_0]\right\}$$

となる。

$t=0$ のとき $0\leqq x\leqq x_0$ に対し

$$\frac{\partial Y}{\partial x}=(1+\gamma)(m+1)Y/(n+2)x$$

$x=x_0$ に対し $\qquad Y=1/(n+1)$

とすれば $0\leqq x\leqq x_0$ に対し

$$Y=\sum_{n=1}C_n\left\{(x/x_0)^{(1+\gamma)(m+1)/(n+2)}\cos[(1+\gamma)(m+1)(n+1)\omega_nt/(n+2)D_1]\div(n\right.$$
$$\left.+1)\sum_{n+1}C_n+J_0[(1+\gamma)(m+1)\omega_nx]\sin[(1+\gamma)(m+1)(n+1)\omega_nt/(n+2)D_1]\right\}$$

となる。

$t=0$ のとき

$$\frac{\partial Y}{\partial t}=(1+\gamma)(m+1)C_1(1-\overline{n+1}Y)/(n+2)D_1x$$

とすれば

$$\sum C_n\omega_nJ_0[(1+\gamma)(m+1)\omega_nx]=\left\{1-(x/x_0)^{(1+\gamma)(m+1)/(n+2)}\right\}/(n+1)x$$

となる。故に

$$C_n=2\int_0^{x_0}\left\{1-(x/x_0)^{(1+\gamma)(m+1)/(n+2)}\right\}J_0[(1+\gamma)(m+1)\omega_nx]dx/(n+1)\omega_nJ_1(j_{on})^2$$

16), 17) により

$$v_1=(1+\gamma)(m+1)\bar{y}g_1(\eta)/(n+2)x,$$

$$\mu_{11}=\{C_1\tau_1(\eta)+D_1x_1(\eta)\}\zeta_0{}^{(1+\gamma)m'}x^{(1+\gamma)(m+1)}/xf_1'(\eta)\bar{y}^n$$

となる。

これらを 9) に代入して積分すれば p_1 が得られる。次に

$$w_1=\bar{y}^2h_1(\eta)(z-z_0)^2/\zeta_0$$

と置けば 11) から w_1 を求めることが出来る。

p, w は $\bar{y}^2{}_s$ の大さであるから一般に省略することが出来る。

更に精密な計算には

$$u_2'=u_2+u_3',v_2'=v_2+v_3',w_2'=w_2+w_3',p_2'=p_2+p_3',\mu_{12}'=\mu_{12}+\mu_{13}'$$

と置き 2), 6) では $\bar{y}^2$ の大さの項と他の項に分け 3), 8) では $\bar{y}^3$ の大さの項と他の項に分け 4), 10) では $\bar{y}^4$ の大さの項と他の項に分けて同様に u_2, v_2, w_2, p_2, μ_{12} を求めることが出来る。この際 u_2 は $y=\bar{y}$ における

$$-p=(u^2+v^2+w^2)/2-1/2$$

の関係を使つて求められる。

いろいろの M の値に対し m, n, m', c_1, d_1, x_0, $\bar{x}_0$, $f_1(\eta)$ を適当に選べば対応する流に対する近似解が得られる。γ を変化して平板表面の状態による流の変化を示すことが出来る。

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